

Optimal Flight Control Synthesis via Pilot Modeling

David K. Schmidt*

Purdue University, West Lafayette, Ind.

The paper includes the development of a synthesis approach usable in the absence of quantitative aircraft handling qualities specifications, and yet explicitly includes design objectives based on pilot-rating concepts by means of an optimal-control pilot model. The methodology uses the pilot's objective function (from which the pilot model evolves) to design the stability augmentation system (SAS). The procedure involves simultaneously solving for the stability augmentation system gains and pilot model via optimal-control techniques. Simultaneous solution is required in this case, since the pilot model (gains, etc.) depends on the augmented plant dynamics, and the augmentation is obviously not known a priori.

Nomenclature

A	= plant system dynamic matrix ($n \times n$)
B	= plant control matrix ($n \times m$)
B_0	= partitioned matrix $\begin{bmatrix} 0 \\ \dots \\ I \end{bmatrix}$ ($(n+m) \times m$)
C	= system observation matrix ($l \times n$)
E	= expected value operator
F	= SAS control weighting matrix in J_{SAS} ($m \times m$)
f	= scalar SAS control weighting
G	= pilot control-rate weighting matrix in J_p ($m \times m$)
I	= identity matrix of appropriate dimension
J	= objective function for SAS or pilot model
K	= Riccati gains—solution to Riccati equation; controlled element gain in example
l	= order of observation vector y
m	= order of control vector u
n	= order of state vector x
P	= augmented state weighting matrix in J_{SAS} ($(n+m) \times (n+m)$)
PR	= pilot rating (Cooper-Harper scale)
Q	= output weighting matrix in J_p ($l \times l$)
R	= pilot-control weighting matrix in J_p ($m \times m$)
u	= control vector, SAS or pilot
V	= observation or motor noise covariance matrix ($l \times l$) or ($m \times m$)
v	= observation or motor noise vector
W	= disturbance noise covariance matrix ($n \times n$)
w	= disturbance noise vector
x	= system state vector
y	= observation vector
β	= dummy variable in least-mean-square predictor
δ	= impulse function; control deflection in example
ϵ	= estimation error vector
θ	= output variable or commanded variable in example
Σ	= estimation error covariance matrix
τ	= observation delay or neuromuscular lag time constant, seconds
x	= augmented state vector

Subscripts

c	= commanded variable
m	= pertaining to neuromotor parameters (noise)
N	= neuromuscular parameters
p	= pertaining to the pilot
SAS	= pertaining to the augmentation system
y	= pertaining to the output parameters

Introduction

FOR flight vehicles with "conventional" dynamic characteristics (i.e., dynamics dominated by the conventional rigid-body modes such as the short-period mode), a host of pilot opinion ratings have been correlated with the damping and frequencies of these modes to yield quantitative handling qualities specifications.¹ These acceptable dynamic properties, or specifications, then provide the flight control designer with quantitative objectives for system synthesis on future vehicles.

However, pilot rating data or quantitative handling qualities specifications are almost nonexistent for flight vehicles exhibiting nonconventional dynamic characteristics resulting from completely foreign operating environments and flight conditions, or radically new aerodynamic and flight-control designs, that is, V/STOL and control-configured vehicles (CCV's). Furthermore, higher-order system dynamics, such as structural/aeroelastic dynamics or even augmentation itself, have been found to significantly alter pilot opinion ratings.^{2,3} In these latter cases, then, the existing handling qualities are inappropriate for use with these high-order systems.

Consequently, the synthesis problem, already made difficult by the complex, multivariable nature of the systems cited above, is compounded by the fact that quantitative design objectives ultimately based on pilot acceptability are unavailable. In this paper, the development of an augmentation synthesis methodology applicable in the absence of handling qualities specifications will be presented. Furthermore, the method explicitly includes design objectives based on pilot acceptability and is clearly applicable to high-order systems.

Background

Two previous methods considered appropriate for high-order systems are the methods of reduced-order or equivalent systems⁴ and model-following.⁵ However, in the absence of prior handling qualities information, the case being considered here, the equivalent systems method is useless for the synthesis problem. With regard to model-following methods, again the system designer has no a priori knowledge of

Received May 24, 1978; presented as Paper 78-1286 at the AIAA Guidance and Control Conference, Palo Alto, Calif., Aug. 7-9, 1978; revision received Sept. 14, 1978. Copyright American Institute of Aeronautics and Astronautics, Inc., 1978. All rights reserved.

Index categories: Guidance and Control; Handling Qualities, Stability and Control.

*Assistant Professor, School of Aeronautics and Astronautics, Associate Fellow AIAA.

desirable system dynamics, hence no information on what would constitute a desirable model to follow. So these methods are considered unapplicable in this case.

In the absence of pilot opinion data, some sort of prediction of pilot rating is required, and any prediction method always includes some form of pilot model. Two types of pilot control models exist, describing function models⁶ and optimal control models (OCM's).⁷ The first analytical pilot-rating-prediction method was Anderson's "paper pilot."⁸ In this approach, parameters in a pilot describing function of assumed form were chosen to minimize a pilot-rating metric. The metric included a measure of closed-loop performance (i.e., rms tracking error) and a measure of pilot work load (in the amount of lead introduced into the system by the pilot). Hollis⁹ applied this methodology to determine the desired augmentation for an aircraft in a longitudinal pitch-tracking task.

However, a drawback of any method based on a pilot describing function is that the pilot's equalization (or form of describing function) must necessarily be assumed a priori. The pilot is known to adapt his gain and form of equalization to the plant dynamics and the piloting task. Prior selection of a pilot describing function for nonconventional vehicle dynamics and new tasks is therefore undesirable. The form or the pilot's equalization, gains, etc., should ideally evolve through the synthesis methodology.

With these facts in mind, the form of the OCM appears ideal. With proper selection of parameters, the OCM predicts both the pilot's gains and equalization. Furthermore, Hess^{10,11} has shown that the optimal control model can also yield pilot opinion predictions for a variety of plants and tasks. Use of this promising method includes a natural pilot-rating metric via the pilot-model objective function. Finally, the optimal-control formulation is most compatible with high-order systems. A summary of this pilot modeling approach is necessary at this point.

The Pilot Model

As presented in Ref. 7, the optimal pilot model evolves from the assumption that the well-trained, well-motivated pilot selects his control input(s) u_p , subject to human limitations, such that the following objective function is minimized:

$$J_p = E \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (y' Q y + u_p' R u_p + \dot{u}_p' G \dot{u}_p) dt \right]$$

The dynamic system being controlled by the pilot is described by the familiar linear relation

$$\begin{aligned} \dot{x} &= A_p x + B_p u_p + w \\ y &= Cx \end{aligned} \quad (1)$$

where w is the vector of zero-mean external disturbances with covariance

$$E[w(t)w'(t+\sigma)] = W\delta(\sigma)$$

Included as human limitations are observation delay τ and observation noise v_y . So the pilot actually perceives the noise-contaminated delayed states, or

$$y_p = C_p x(t-\tau) + v_y(t-\tau)$$

The covariance of the zero-mean observation noise may include the effects of perception thresholds and attention allocation, and is denoted

$$E[v_y(t)v_y'(t+\sigma)] = V_y\delta(\sigma)$$

Defining $\chi = \text{col}[\hat{x}, u_p]$, the solution to the problem, or the pilot's control, is given as

$$\dot{u}_p^* = -G^{-1}[0 \mid I]K_p\chi$$

where K_p is the positive definite solution to the Riccati equation

$$\begin{aligned} - \left[\begin{array}{c|c} A_p & B_p \\ \hline 0 & 0 \end{array} \right]' K_p - K_p \left[\begin{array}{c|c} A_p & B_p \\ \hline 0 & 0 \end{array} \right] \\ - \left[\begin{array}{c|c} C_p' Q C_p & 0 \\ \hline 0 & R \end{array} \right] + K_p B_p G^{-1} B_p' K_p = \dot{K}_p \end{aligned} \quad (2)$$

It will be convenient to partition K_p such that

$$K_p = \left[\begin{array}{c|c} K_{p1} & K_{p2} \\ \hline K_{p3} & K_{p4} \end{array} \right]$$

and note that now the equation for the optimal control u_p^* is

$$\dot{u}_p^* = -G^{-1}K_{p3}\hat{x} - G^{-1}K_{p4}u_p^*$$

or a linear feedback of the best estimate of the state \hat{x} and some control dynamics. (These control dynamics have been shown to be equivalent to the pilot's neuromuscular lag.)

To model the pilot's remnant, motor noise is usually added to the control equation. The final pilot's control is represented by

$$\dot{u}_p^* = -G^{-1}K_{p3}\hat{x} - G^{-1}K_{p4}u_p^* + v_m$$

where

$$E[v_m(t)v_m'(t+\sigma)] = V_m\delta(\sigma)$$

Now, the state estimator consists of a Kalman filter and a least-mean-square predictor, or

$$\begin{aligned} \dot{\hat{x}}(t-\tau) &= A_p\hat{x}(t-\tau) + \Sigma C_p' V_y^{-1} [y_p(t) \\ &\quad - C_p\hat{x}(t-\tau)] + B_p u_p^*(t-\tau) \end{aligned}$$

$$\hat{x}(t) = \beta(t) + e^{A_p\tau} [\hat{x}(t-\tau) - \beta(t-\tau)]$$

with

$$\dot{\beta} = A_p\beta + B_p u_p^*$$

and Σ is the solution of the Riccati equation

$$A_p\Sigma + \Sigma A_p' + W - \Sigma C_p' V_y^{-1} C_p \Sigma = [0]$$

This system of equations, when solved, determines the optimal-control pilot model.

Finally, as noted previously, Hess^{10,11} has found that when the weightings on the state and control (i.e., Q and R) in the pilot's objective function are appropriately selected, the resulting magnitude of the pilot's objective function, after solving for the pilot model, is strongly correlated with the pilot's rating of the vehicle and task. If the pilot rating is given in the Cooper-Harper system, and if the results in Refs. 9 and 10 are reduced to a regression equation, the relation is

$$\text{Pilot Rating (PR)} = 25 \cdot \ln(10 J_p) + 0.3$$

Now, through this relation and the solution of the pilot model above, we have not only a pilot-control model but a prediction of the pilot's rating of the dynamic system.

Augmentation Synthesis Method

In the determination of the pilot model parameters above, we have expressed the system dynamics in terms of the matrices A_p and B_p . However, since the augmentation has not been defined, the augmented plant, A_p and B_p , is as yet unknown.

Consider the unaugmented plant dynamics to be described by

$$\dot{x} = Ax + Bu + w$$

However, A and B are now the unaugmented system matrices, and u is the control input vector. Now, the total control input to the plant will include pilot input u_p plus augmentation input u_{SAS} , or

$$u = u_p + u_{SAS}$$

Further, from the pilot model, we know that although the feedback gains (e.g., $G^{-1}K_{p3}$, $G^{-1}K_{p4}$) have not been determined, the pilot's control input is expressible as

$$\dot{u}_p = -G^{-1}K_{p3}\hat{x} - G^{-1}K_{p4}u_p$$

Now, the estimate of the state, \hat{x} , can be expressed in terms of the true state plus some estimation error ϵ , or

$$\hat{x} = x + \epsilon$$

By treating this error as another disturbance w_p , we can write the pilot's control equation as

$$\dot{u}_p = -G^{-1}K_{p3}x - G^{-1}K_{p4}u_p + w_p \quad (3)$$

(Note, the disturbance term w_p can also include the pilot's remnant as well.) Combining this relation with the plant-dynamic and control equations we have

$$\dot{\chi} = \begin{bmatrix} A & B \\ -G^{-1}K_{p3} & -G^{-1}K_{p4} \end{bmatrix} \chi + \begin{bmatrix} B \\ 0 \end{bmatrix} u_{SAS} + \begin{bmatrix} w \\ w_p \end{bmatrix} \quad (4)$$

where $\chi = \text{col}[x, u_p]$.

We now may proceed to determine an objective function for determining u_{SAS} .

From the correlation between pilot rating and the pilot's objective function, we clearly see that the best (i.e., lowest) pilot rating implies the lowest pilot objective function. Therefore, for optimum pilot rating, the control u_{SAS} should be chosen to minimize J_p as defined in the pilot rating method.¹⁰ (This method defines the state and control weights, Q and R , as the inverse of the maximum allowable deviations in the variables as perceived by the pilot.) Finally, to preclude infinite augmentation gains, we must also penalize augmentation control energy. Therefore, the augmentation is chosen to minimize

$$J_{SAS} = J_p + E \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u'_{SAS} F u_{SAS} dt \right]$$

or

$$J_{SAS} = E \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (y' Q y + u'_p R u_p + \dot{u}'_p G \dot{u}_p + u'_{SAS} F u_{SAS}) dt \right]$$

and Q , R , and G are as chosen in the pilot's objective function J_p .

This may be written as

$$J_{SAS} = E \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T (\chi' P \chi + u'_{SAS} F u_{SAS}) dt \right]$$

where

$$P = \begin{bmatrix} C'_p Q C_p + K'_{p3} G^{-1} K_{p3} & K'_{p3} G^{-1} K_{p4} \\ K'_{p4} G^{-1} K_{p3} & R + K'_{p4} G^{-1} K_{p4} \end{bmatrix}$$

and instead of Eq. (3) being substituted for \dot{u}_p in the above J_{SAS} , we have invoked a form of separation principle and substituted the relation

$$\dot{u}_p = -G^{-1}K_{p3}x - G^{-1}K_{p4}u_p$$

The heuristic justification for the use of this relation lies in the fact that we wish to synthesize the augmentation on the basis of how the pilot is *trying* to perform the control function rather than on how the pilot is capable of doing so. The effect is to eliminate the accuracy of the pilot's state estimate from the augmentation synthesis. We are concerned here with system-dynamic modification rather than display design or the pilot's estimation process.

With this objective function and the system dynamics given in Eq. (4), the problem is now stated in conventional form, except K_{p3} and K_{p4} are as yet undetermined of course. If we assume, for example, full-state feedback, the solution of this problem is known to be

$$u^*_{SAS} = -F^{-1} [B' \mid 0] K_{SAS} x$$

or

$$u^*_{SAS} = -F^{-1} B' K_{SAS_1} x - F^{-1} B' K_{SAS_2} u_p$$

where

$$K_{SAS} = \begin{bmatrix} K_{SAS_1} & K_{SAS_2} \\ K_{SAS_3} & K_{SAS_4} \end{bmatrix}$$

is the solution to the Riccati equation

$$\begin{aligned} & - \begin{bmatrix} A & B \\ -G^{-1}K_{p3} & -G^{-1}K_{p4} \end{bmatrix}' K_{SAS} \\ & - K_{SAS} \begin{bmatrix} A & B \\ -G^{-1}K_{p3} & -G^{-1}K_{p4} \end{bmatrix} \\ & - P + K_{SAS} \begin{bmatrix} B \\ 0 \end{bmatrix} F^{-1} [B' \mid 0] K_{SAS} = \dot{K}_{SAS} \end{aligned} \quad (5)$$

We see in this expression that the solution for K_{SAS} obviously depends on K_p (or K_{p3} and K_{p4}). Returning to the Riccati equation (2) for the pilot gain, we also see that that equation depends in turn on the SAS gains (or K_{SAS}), since the pilot Riccati equation involves the *augmented* plant matrices A_p and B_p . As a result of the SAS design procedure just presented, we now know, however, the SAS structure. Returning to the pilot model, we may now include this SAS structure specifically, so that A_p and B_p , as in Eq. (1), may in fact be expressed as

$$A_p = A - B F^{-1} B' K_{SAS_1}$$

and

$$B_p = B(I - F^{-1} B' K_{SAS_2})$$

Substituting these expressions in the pilot Riccati equation yields two coupled Riccati equations, one for the pilot gains, Eq. (2), and one for the SAS gains, Eq. (5). These may be

solved simultaneously for K_{SAS} and K_p by integrating both equations backward. Note that this solution does not involve a two-point boundary-value problem. Finally, although optimality of the solution has not been formally addressed here, in the author's experience to date the integration has converged to the positive-definite unique solutions.

A Simple Numerical Example

Consider a simple tracking task with the controlled element (plant) dynamics considered in Ref. 10:

$$\theta(s)/\delta(s) = K/s^2 \quad (K=11.7)$$

The command signal θ_c is white noise w , passed through the filter:

$$\theta_c(s)/w(s) = 3.67/(s^2 + 3s + 2.25)$$

and

$$E(w) = 0 \quad E[w(t)w(t+\sigma)] = \delta(\sigma)$$

If we define the state vector as $x = \text{col}(\theta_c, \dot{\theta}_c, \theta, \dot{\theta})$, we have the plant

$$\dot{x} = Ax + B\delta + 3.67w$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.25 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B' = [0, 0, 0, 11.7]$$

For this system, error and error rate are perceived by the pilot or

$$y_p = \begin{bmatrix} \theta_c & -\theta \\ \dot{\theta}_c & -\dot{\theta} \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & 0 & -1 \end{bmatrix} x$$

The performance index, chosen consistent with Hess' rating hypothesis, is

$$J_p = E \left\{ \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T [\theta_c - \theta]^2 + 0.01\delta_p^2 + g\dot{\delta}_p^2 dt \right\}$$

and g is chosen to yield a neuromuscular lag, $1/\tau_N = G^{-1}K_{p4} = 10$, or $\tau_N = 0.1$ seconds.

In the above expression, the form of the pilot's objective function, J_p , to be used deserves some discussion. In the references cited, Hess used a J_p without the quadratic penalty on control rate, or $\dot{u}_p' G \dot{u}_p$. However, recent work by Hess (as yet unpublished) has shown that use of this form in the pilot-rating metric may lead to erroneous rating predictions for various control (stick) sensitivities (i.e., B). This phenomenon is less pronounced if the complete J_p , as defined previously in the pilot model, is used in the above metric. Furthermore, use of a J_p without the control penalty $u_p' R u_p$, but including a control rate penalty and varying the weighting G to yield the effect of a constant neuromuscular lag, appears to completely eliminate the stick sensitivity issue. However, since more research appears needed in this area, in this presentation we will use the complete J_p as defined in the pilot model. Should later work indicate that a different form is appropriate, this adjustment will be made at that time.

Unaugmented, the pilot Riccati equations are solved with the following noise statistics (human limitations):

- 1) Equal, attentional allocation, 50% for both error and error rate.
- 2) Observation thresholds on error and error rate = 0.5 (units of display displacement).
- 3) Sensor noise $(V_y)_{ii}/E(y_i^2) = -20$ dB ($i=1,2$).
- 4) Motor noise $(V_u)_{ii}/E(u_i^2) = -20$ dB, where $u_c = -G^{-1}K_{p3}\hat{x}$.
- 5) Observation delay $\tau = 0.1$ seconds.

The "piloted" system performance is given in Table 1.

Assuming full-state feedback, the augmentation control law is

$$\delta_{SAS} = -K_1\theta_c - K_2\dot{\theta}_c - K_3\theta - K_4\dot{\theta} - K_\delta\delta_p$$

The SAS objective function is

$$J_{SAS} = J_p + E \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T \delta_{SAS}^2 dt \right]$$

So the piloted plant, including augmentation, will be

$$\dot{x} = A_p x + B_p \delta_p + 3.67w$$

where

$$A_p = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -2.25 & -3 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -11.7K_1 & -11.7K_2 & -11.7K_3 & -11.7K_4 \end{bmatrix}$$

$$B_p' = [0, 0, 0, 11.7(1-K_\delta)]$$

Solving the pilot and SAS Riccati equations simultaneously and then determining piloted system performance as before yield the results given in Table 2. The augmentation gains, $K_1 - K_4$ and K_δ , for the three cases above are given in Table 3, along with the augmented plant eigenvalues.

The agreement between the trend in plant dynamics resulting from augmentation and the short-term longitudinal response requirements, Fig. 1 of Ref. 12, is shown in Fig. 1.

Table 1 Unaugmented system performance

$(\theta_c - \theta)$ rms	δ_p rms	J_p	PR ^a
1.17	1.00	1.86	7.7

^a Predicted pilot rating based on J_p .

Table 2 Augmented system performance

f	$(\theta_c - \theta)$ rms	δ_p rms	J_p^a	PR ^b
100.0	1.10	0.89	1.60	7.3
10.0	0.79	0.61	0.77	5.4
1.0	0.38	0.35	0.17	1.6

^a Note this is the numerical value of the pilot's objective function, J_p , not J_{SAS} .

^b Predicted pilot rating based on J_p .

Table 3 Augmentation gains

f	K_1	K_2	K_3	K_4	K_5	Plant eigenvalues ^a
100	-0.009	-0.002	0.009	0.003	0.004	$-0.017 \pm 0.331j$
10	-0.078	-0.016	0.084	0.024	0.036	$-0.142 \pm 0.982j$
1	-0.513	-0.090	0.542	0.130	0.155	$-0.758 \pm 2.400j$

^aNot including noise filter eigenvalues of course.

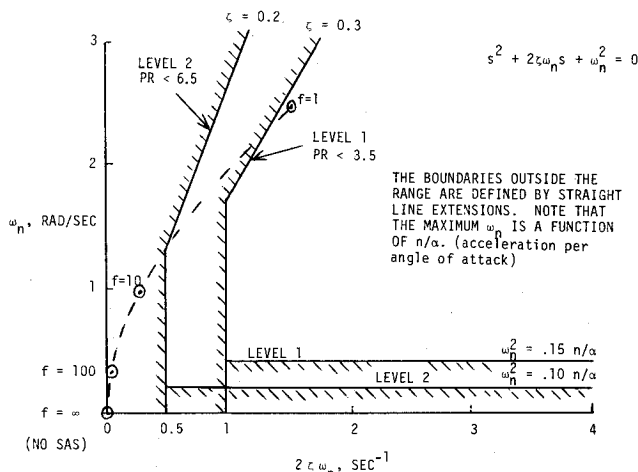


Fig. 1 Short-term longitudinal response requirements.

Conclusions

In summary, we have cited the flight-dynamic and control problems of nonconventional flight vehicles (V/STOL and CCV) due to the complexity of augmentation required and the lack of handling qualities objectives. We have presented a methodology intended to be suitable for this type of problem. The method uses an optimal control pilot model to predict not only piloted performance but pilot rating as well. With the optimal-control model structure, we were able to formulate the augmentation synthesis problem as an optimal-control problem with the parameters in plant matrices depending on the pilot model, and vice versa. This necessitates simultaneous solution of the two (pilot and augmentation) problems. We have included the form of the solution under the assumption of full-state variable feedback and no measurement noise, and a simple numerical example.

The first logical extension is the solution for the case of limited state feedback. This limited feedback case is actually closer to pure plant augmentation than the case addressed here. In our solution and in the example, we have closed the tracking loop, and pure plant augmentation would only feedback plant states. However, the primary purpose of our

discussion here was to provide the problem structure that would be unchanged regardless of augmentation approach.

Acknowledgments

The author would like to express his appreciation to the referees for their helpful suggestions and to R. Hess for a number of fruitful discussions and comments.

References

- ¹"Military Specification—Flying Qualities of Piloted Airplanes," MIL-F-8785B (ASG), 1969.
- ²Neal, T.P. and Smith, R.E., "An In-Flight Investigation to Develop Control System Design Criteria for Fighter Airplanes," AFFDL-TR-70-74, Vols. 1 and 2, Dec. 1970.
- ³Swaim, R.L. and Yen, W.Y., "Effects of Dynamic Aeroelasticity on Handling Qualities and Pilot Rating," AIAA Paper 78-1365, 5th Atmospheric Flight Mechanics Conference, Palo Alto, Calif., Aug. 1978.
- ⁴Hodgkinson, J. and LaManna, W.J., "Equivalent System Approaches to Handling Qualities Analysis and Design Problems of Augmented Aircraft," AIAA Paper 77-1122, 4th Atmospheric Flight Mechanics Conference, Hollywood, Fla., Aug. 1977.
- ⁵Kriechbaum, G. K.L. and Stineman, R.W., "Design of Desirable Airplane Handling Qualities via Optimal Control," *Journal of Aircraft*, Vol. 9, May 1972, pp. 365-369.
- ⁶McRuer, D.T. and Krendel, E.S., "Mathematical Models of Human Pilot Behavior," AGARD-AG-188, AGARD, NATO, Jan. 1974.
- ⁷Kleinman, D.L., Baron, S., and Levison, W.H., "An Optimal Control Model of Human Response, Pt. I: Theory and Validation, Part II: Prediction of Human Performance in a Complex Task," *Automatica*, Vol. 6, 1970, pp. 357-383.
- ⁸Anderson, R., "A New Approach to the Specification and Evaluation of Flying Qualities," USAF Flight Dynamics Laboratory, AFFDL-TR-69-120, 1969.
- ⁹Hollis, T.L., "Optimal Selection of Stability Augmentation System Parameters to Reduce the Pilot Ratings for the Pitch Tracking Task," Air Force Institute of Technology, Wright Patterson Air Force Base, Ohio, M.S. Thesis GGC/EE/71-10, June 1971.
- ¹⁰Hess, R.A., "A Method of Generating Numerical Pilot Opinion Ratings Using the Optimal Pilot Model," NASA-TMX-73, 101, Feb. 1976.
- ¹¹Hess, R.A., "Prediction of Pilot Opinion Rating Using an Optimal Pilot Model," *Human Factors*, Vol. 19, Oct. 1977, pp. 459-475.
- ¹²"Military Specification—Flying Qualities of Piloted V/STOL Aircraft," MIL-F-83300, 1970, p. 19.